Outage probability of beamforming for multiuser MIMO relay networks with interference

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Ergodic Mutual Information and Its Fluctuation in Multi-Level MIMO Relay System

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The help of relay devices is of paramount importance to overcome the impairments of channel randomness, especially in a rich scattering scenario with no line-of-sight communication path. Relaying strategies are gaining increasing attention due also to the eventual deployment of small-cell-based networks, where short-range communication is inherently to be favored. MIMO is naturally combined with relaying in order to further increase the overall spatial degrees of freedom of the system. Since the early introduction of relay-aided transmission protocols, in the field of information theory lot of efforts has been devoted to the analytical characterization of the performance of such strategies, with a main focus on dual-hop (see e.g. [1, 2] and refs. therein) i.e. single relay-stage systems.

In a generic multi-hop relay communication channel the trade-off between system parameters and spectral and/or energy efficiency have yet to be fully unveiled, but for some preliminary results in the pure SISO case [3]. Therein, it is shown that the error exponent of such a channel is not monotonic in the number of hops, hence motivating the optimization of the number of hops. An analytical characterization of multi-hop MIMO relay systems is only available in the large system limit, e.g., as the number of antennas at the source, at the destination and at each relay level grows unbounded with prescribed reciprocal constraints [4, 5]. Relying on very recent results from random matrix theory and polynomial ensembles [6], in this work we move a step toward a full characterization of the multi-hop relay system for a finite number of antennas at every device, non-noisy relays and white noise at the destination. The results are valid for arbitrary signal-to-noise ratios (SNR). Moreover we assume communication channels affected by uncorrelated Rayleigh fading, and that only the destination is provided with statistical channel state information (CSI). We provide exact closed-form expressions, for both the average mutual information as well as for its variance, thus characterizing also mutual information main fluctuations. Results are provided in terms of Meijer G-functions of a single [7] and two variables [8]. Our main results are as follows.

Theorem *Let $\mathbf{y} = \widetilde{\mathbf{H}}\mathbf{x} + \mathbf{n}$ be a linear system where $\mathbf{y}$ is a vector of size M, $\mathbf{x}$ is a vector of size-K, $\mathbf{n}$ represents additive white gaussian noise, and the random channel matrix, $\widetilde{\mathbf{H}}$, has the following expression*

$$\widetilde{\mathbf{H}} = \prod_{i=1}^{N} \eta_i \mathbf{H}_i \,.$$

where $\mathbf{H}_i$ is a $(K + \nu_i) \times (K + \nu_{i-1})$ random matrix with i.i.d. Gaussian entries and $\nu_0 = 0$. The matrix $\widetilde{\mathbf{H}}$ models a N-hops amplify-and-forward MIMO relay system with uniform power allocation (UPA) at the source and at each relay stage, with amplification factors η_i, negligible noise level at the relays, and noisy receiver at the destination.

Then the first and second moments of the mutual information

$$\mathcal{I}(\eta) = \log\det(\mathbf{I}_K + \eta\widetilde{\mathbf{H}}\widetilde{\mathbf{H}}^{\mathsf{H}})$$

are given by, respectively,

$$E[\mathcal{I}(\eta)] = c \sum_{i,j=1}^{K} \alpha_{i,j} G^{N+2,2}_{1,N+2}\left(\begin{array}{l} 1,1,1-j-\nu_N,\ldots,1-j-\nu_2,2-(i+j)-\nu_1 \\ 1,0 \end{array} |\eta \right)$$

and

$$E[\mathcal{I}^2(\eta)] = c \sum_{i,j=1}^{K} \alpha_{i,j} G^{0,N;1,2;1,2}_{N,0;2,2;2,2}\left(\begin{array}{l} 1-j-\nu_N,\ldots,2-i-j-\nu_1 \\ 1-j,\ldots,1-j \end{array} \middle| \begin{array}{l} 1,1 \\ 1,0 \end{array} \middle| \begin{array}{l} 1,1 \\ 1,0 \end{array} \middle| \eta,\eta \right).$$

In the above expressions, we employed the Meijer G-function of a single and two variables (see [7, 8] for details), we set $\eta = \prod_{i=1}^{N} \eta_i$, with $\eta_N = 1$, as the overall transmitted power (through all relay stages). Moreover, $\alpha_{i,j}$ is the (i,j)-th co-factor of a matrix $\mathbf{A}$ whose (k,h)-th entry is given by $a_{k,h} = \Gamma(k+\nu_1+h-1)\prod_{i=2}^{N}\Gamma(h+\nu_i)$. Finally, c is a normalizing constant.

Being UPA optimal in absence of transmit CSI, this analysis provides closed-form expressions for the average achievable rate in a realistic setting where CSI sharing is limited due to the need to reduce time/energy consumption. Further results for the case in which also the destination is unaware of CSI are not reported in this abstract due to lack of space but will be reported in the extended version of the work.

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